New t-designs and large sets of t-designs

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Abstract

Recent results in the search for simple t-designs and large sets of t-designs are reported. Many new parameter sets of simple t-designs on up to 40 points and $t \geq 7$ are given. The tool used is a program DISCRETA, developed by the authors, which applies the method of Kramer-Mesner [15] where an automorphism group of the desired designs is prescribed. In several cases Tran van Trung's [28] and Alltop's [2] construction yield further results from those found by computer. By computer search we also constructed new large sets of t-designs, which enable via a theorem by Ajoodani-Namini [1] the construction of infinite families of large sets of t-designs.

Key words: t-designs, large sets of t-designs, group actions, constructive combinatorics

1 Introduction

A simple t-(v, k, λ) design $\mathcal{D}$ is defined as a set of k-subsets, called blocks, of a set V of v points such that each t-subset of V is contained in exactly λ blocks. Since we only consider simple designs, we omit the word simple.

A *large set* of t-(v, k, λ) designs, denoted by $\mathrm{LS}[n](t, k, v)$, is a partition of the complete design, i.e. the set of all k-subsets of V, into n disjoint t-(v, k, λ) designs. It follows that $\lambda = \binom{v-t}{k-t}/n$.

Most designs with small parameter sets and "large" t were found using Kramer-Mesner matrices. We also follow this approach and, like D. L. Kreher and S. P. Radziszowski [17], use an LLL-algorithm for solving systems of linear Diophantine equations [30]. Our program system DISCRETA allows to choose a permutation group from several families like projective linear groups and

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make some group constructions. The resulting group is then prescribed as an automorphism group of the desired designs. Any such design is a collection of full orbits of that group on the set of k-subsets of V. Finding a collection which forms a t-design is equivalent to the problem of solving a system of linear Diophantine equations.

In some cases the constructed designs are part of a large set $\mathrm{LS}[n](t, k, v)$ of t-designs. If n is prime the method of Ajoodani-Namini [1] can be applied and an infinite family of large sets of t-designs can be constructed.

2 Combining designs and large sets of designs

There are several ways known on how to construct new parameter sets from an existing set of designs or a large set of designs:

2.1 From t- to $(t-1)$-designs

It is well known (see for example [16]) that a given t-design leads to 3 different types of $(t-1)$-designs. Their parameter sets can be deduced as follows:

$$t\text{-}(v, k, \lambda_t) \rightarrow \begin{cases} (t-1)\text{-}(v, k, \lambda_{(t-1)}), & \text{the reduced design,} \\ (t-1)\text{-}(v-1, k-1, \lambda_t), & \text{derived designs,} \\ (t-1)\text{-}(v-1, k, \lambda_{(t-1)} - \lambda_t), & \text{residual designs,} \end{cases}$$

where for $\lambda \in \mathbb{N}$, λ_i is defined through $\lambda_i = \lambda\binom{v-i}{t-i}/\binom{k-i}{t-i}$, $0 \leq i \leq t$, i.e. $\lambda_t = \lambda$.

2.2 From t- to t-designs

The complement of a t-(v, k, λ) design is a t-$(v, k, \binom{v-t}{k-t} - \lambda)$ design, which we will not mention in the sequel. Tran van Trung constructs from two $(t-1)$-designs another $(t-1)$-design:

Theorem 1 (Tran van Trung [28]) *If there exist a $(t-1)$-$(v-1, k-1, \lambda_t)$ and a $(t-1)$-$(v-1, k, \lambda_{(t-1)} - \lambda_t)$ design, then also a $(t-1)$-$(v, k, \lambda_{(t-1)})$ design exists.*

Even large sets can be combined to give new large sets: A construction like Theorem 1 for large sets of designs is

Theorem 2 (Khosrovshahi, Ajoodani-Namini [12]) *If a* $LS[n](t,k,v)$ *and a* $LS[n](t,k+1,v)$ *exist, then a* $LS[n](t,k+1,v+1)$ *exists.*

2.3 From t*- to* $(t+1)$*-designs*

Alltop constructs $(t+1)$-designs out of t-designs:

Theorem 3 (Alltop [2]) *Let* $\mathcal{D}$ *be a* t-$(2k+1,k,\lambda)$ *design.*

(1) If t *is even, then there exists a* $(t+1)$-$(2k+2,k+1,\lambda)$ *design.*
(2) If t *is odd and* $\lambda = \frac{1}{2}\binom{v-t}{k-t}$, *then there exists a* $(t+1)$-$(2k+2,k+1,\lambda)$ *design.*

In 1996 an analogue of Theorem 3 for large sets of designs appeared:

Theorem 4 (Ajoodani-Namini [1]) *Let* p *be a prime and* v, k, n *be positive integers such that* $np < k < (n+1)p$ *and* $v > n$. *If there exists a* $LS[p](t,n,v-1)$, *then also a* $LS[p](t+1,k,pv)$ *exists.*

2.4 Infinite families

For a long time t-designs were only known for $t \leq 5$. In 1984 Magliveras and Leavitt [19] constructed the first 6-designs using the method of Kramer-Mesner [15] and a computer search.

In 1987 Teirlinck proved that t-$(v,t+1,\lambda)$ designs exist for all t:

Theorem 5 (Teirlinck [26]) *Given integers* t, v *with* $v \equiv t \bmod (t+1)!^{2t+1}$ *and* $0 < t+1 \leq v$, *then a simple* t-$(v,t+1,(t+1)!^{2t+1})$ *design exist.*

The proof is constructive, but the resulting designs have extremely large parameters. Thus, small examples and cases where k is greater than $t+1$ are still interesting. A recent review can be found in the Handbook of Combinatorial Designs [16].

In 1989, Teirlinck [27] showed the existence of large sets of t-designs for all values of t and n:

Theorem 6 *For every natural number* t *let* $\lambda(t) = \mathrm{lcm}(\binom{t}{m} \mid m = 1,2,\ldots,t)$, $\lambda^*(t) = \mathrm{lcm}(1,2,\ldots,t+1)$ *and* $\ell(t) = \prod_{i=1}^{t} \lambda(i) \cdot \lambda^*(i)$. *Then for all* $n > 0$, *there is a* $LS[n](t,t+1,t+n\cdot\ell(t))$.

Ajoodani-Namini constructed another infinite family:

Theorem 7 (Ajoodani-Namini [1]) *For integers $t \geq 6$ and $m \geq 2$ a $LS[2](t, 2^{t-3}-1, m \cdot 2^{t-3}-2)$ exists.*

The smallest parameter set of a 7-design one can get using Theorem 7 is a 7-$(30, 15, 245\,157)$ design. The smallest parameter set of an 8-design based on Theorem 7 is a 8-$(62, 31, 542\,964\,991\,579\,920)$ design.

For a large set $\mathrm{LS}[2](t, k, v)$ of designs Theorem 4 can be used to produce the following formula for an infinite family:

Proposition 8 *If for $s,\ k,\ v \in \mathbb{N}$ a $LS[2](s, k, v)$ exists, then for every $t \geq s$ also a $LS[2](t, 2^{t-s}(k+1)-1, 2^{t-s}(v+2)-2)$ exists.*

PROOF. Induction on t together with Theorem 4 for $p = 2$. $\square$

3 Construction by computer

We have built a software package DISCRETA with a graphical user interface for computational construction of t-designs and large sets of t-designs. The program is very easy to use, most commands are supplied by pressing some button or choosing menu entries. We briefly list some features of the system.

(1) The prescribed automorphism group can be chosen out of certain families of groups. We mention the families $\mathrm{PSL}(n, q)$, $\mathrm{PGL}(n, q)$, $\mathrm{P\Sigma L}(n, q)$, $\mathrm{P\Gamma L}(n, q)$ of projective linear groups, cyclic groups and their holomorphs, induced actions of symmetric groups. In addition, one can read in permutations generating the desired group from a file or give a set of generators and defining relations for the group and apply a module for low index methods to get the permutation representation needed.
(2) These groups can be manipulated by forming sums, products, wreath products, by adding fixed points or forming stabilizers. So, if a group is chosen the point set together with the corresponding permutation representation is automatically determined.
(3) Now the design parameters t and k can be chosen. The program computes values of λ such that t-(v, k, λ) forms an admissible parameter set.
(4) If the set of parameters looks promising one can start the computation of the Kramer-Mesner matrix. Internally, double coset algorithms are used to construct that matrix. The program is a new implementation of Schmalz's *Leiterspiel* [23] ("snakes and ladders") by A. Betten who also wrote most of the code of the system. It is also possible to use *orderly generation* to build the Kramer-Mesner matrix.

(5) In order to solve the resulting system of linear Diophantine equations two algorithms are included:

A program written by B. D. McKay performs a clever backtrack search and is especially useful in showing that no design with the prescribed parameter values and group exists. Also, at least for smaller values of λ or matrices with only few rows and many columns this program is our best choice.

Alternatively, a program by A. Wassermann [30] with his version of the LLL-algorithm solves the systems of equations even in cases of some hundred rows and columns. The algorithm consists of blockwise Korkine-Zolotarev reduction [24] implemented with the Gaussian volume heuristics [25]. It is based on lattice basis reduction [18] with "deep insertions" [24]. After the lattice basis reduction an exhaustive search is done by l_∞-enumeration [22,30].

Interestingly, in the lattice basis reduction phase this version sometimes finds designs also for values of λ different from the one given as input to DISCRETA.

(6) A database is used for keeping the parameter sets found. This allows to ask for all stored parameter sets where each parameter lies in a given range. Inserting a new parameter set automatically generates the parameter sets of the derived, reduced and residual designs recursively and stores them in the database. Also it tries to apply the construction of Tran van Trung [28]. This way, a lot of parameter sets are easily generated from a few starting parameter sets.

(7) The program computes intersection numbers and generalized intersection numbers of the designs, see Ray-Chaudhuri and Wilson [21], Mendelsohn [20], Köhler [13] and Tran van Trung, Qiu-rong Wu, Mesner [29].

(8) Finally a detailed LaTeX-report of the computation can be produced.

With these algorithms the following new results for $t \geq 6$ were achieved:

There exist 6-$(25, 12, \lambda)$ designs for the following values of λ:

$$\lambda = \begin{array}{r} 7728,\ 7854,\ 8190,\ 8316,\ 8652,\ 8778,\ 9114,\ 9240, \\ 9702,\ 10164,\ 10500,\ 10626,\ 10962,\ 11088,\ 11424,\ 11550, \\ 11886,\ 12012,\ 12348,\ 12474,\ 12810,\ 12936,\ 13272,\ 13398. \end{array} \tag{1}$$

The prescribed automorphism group is PSL$(2, 23)$ together with one fixpoint. The Kramer-Mesner matrix had 41 rows and 906 columns. Using Theorem 3 one gets for each value of λ in (1) a 7-$(26, 13, \lambda)$ design with automorphism group PSL$(2, 23)$ together with two fixpoints.

On 20 points exist 7-$(20,10,126)$ designs with automorphism group $\mathrm{PSL}(2,8)\times C_2$ together with two fixpoints. The size of the Kramer-Mesner matrix is 111×244.

7-$(22,11,\lambda)$ designs with automorphism group $\mathrm{PSL}(2,19)++$ exist for $\lambda = 315, 630$. 7-$(26,11,\lambda)$ designs exist with automorphism group $\mathrm{P\Gamma L}(2,25)$ for $\lambda = 1176, 1356, 1536, 1716, 1896, 1926$. The size of the Kramer-Mesner matrix is 70×252, respectively 34×293.

On 29 points exist a 7-$(29,10,420)$ design with $\mathrm{P\Gamma L}(2,27)+$ as group of automorphisms and 7-$(29,11,\lambda)$ designs with automorphism group $\mathrm{P\Gamma L}(2,27)+$ for $\lambda = 2130, 3465$. The size of the Kramer Mesner matrix is 43×391, respectively 43×647.

On 33 points we found 7-$(33,9,\lambda)$ designs with $\mathrm{P\Gamma L}(2,32)$ as automorphism group for $\lambda = 65, 80, 85, 100, 105, 120, 125, 140, 145, 160$ and 7-$(33,10,\lambda)$ designs with automorphism group $\mathrm{P\Gamma L}(2,32)$ for $\lambda = 600, 720, 840, 880$. The size of the Kramer-Mesner matrix is 32×248, respectively 45×345. 7-$(34,10,\lambda)$ designs with automorphism group $\mathrm{P\Gamma L}(2,32)+$ exist for $\lambda = 135, 171$. The size of the Kramer Mesner matrix is 32×596.

On 36 points there exist 7-$(36,11,\lambda)$ designs for $\lambda = 3360$, 4200, 4536, 4935, 5040, 5271, 5376, 5775, 5880, 6111, 6216, 6615, 6720, 7056, 7455, 7560, 7791, 7896, 8295, 8400, 8631, 8736, 9240, 9471, 9576, 9975, 10080, 10311, 10416, 10815, 10920, 11151, 11655, 11760 with the automorphism group $Sp(6,2)_{36}$. The size of the Kramer-Mesner matrix is 37×694.

There are 6-$(24,12,\lambda)$ designs with automorphism group $\mathrm{PSL}(2,23)$ for values of $\lambda = 6510, 7392, 7896, 8778, 8820, 8862, 9240, 9282$. Most remarkable is the last value $\lambda = 9282$ because this design is a halving of the complete design and therefore gives a $\mathrm{LS}[2](6,12,24)$. Applying Theorem 8 we have the following infinite sequences:

Proposition 9

(1) If $t \geq 6$, then a $LS[2](t,\ 2^{t-6}\cdot 13-1,\ 2^{t-6}\cdot 26-2)$ exists.
(2) If $t \geq 3$, then a $LS[2](t,\ 2^{t-3}\cdot 6-1,\ 2^{t-3}\cdot 14-2)$ exists.
(3) If $t \geq 2$, then a $LS[2](t,\ 2^{t-2}\cdot 7-1,\ 2^{t-2}\cdot 14-2)$ exists.
(4) If $t \geq 4$, then a $LS[2](t,\ 2^{t-4}\cdot 7-1,\ 2^{t-4}\cdot 22-2)$ exists.

PROOF. Computer search with the program DISCRETA showed the existence of a

(1) $\mathrm{LS}[2](6,12,24)$,

(2) LS[2](3, 5, 12) with the symmetry group of the truncated cube as automorphism group,
(3) LS[2](2, 6, 12) with the symmetry group of the icosahedron as automorphism group,
(4) LS[2](4, 6, 20) with the symmetry group of the dodecahedron as automorphism group.

Proposition 8 gives the result. For the parameter sets in (2), (3) and (4) large sets of designs with different automorphism groups are already known, see [10]. □

Large sets of t-designs which are listed in [16] give the following infinite families:

Proposition 10

(1) For all $t \geq 4$ a $LS[2](t,\ 2^{t-4} \cdot 6 - 1,\ 2^{t-4} \cdot 14 - 2)$ exists,
(2) For all $t \geq 4$ a $LS[2](t,\ 2^{t-4} \cdot 6 - 1,\ 2^{t-4} \cdot 30 - 2)$ exists,
(3) For all $t \geq 4$ a $LS[2](t,\ 2^{t-4} \cdot 6 - 1,\ 2^{t-4} \cdot 46 - 2)$ exists,
(4) For all $t \geq 4$ a $LS[2](t,\ 2^{t-4} \cdot 6 - 1,\ 2^{t-4} \cdot 130 - 2)$ exists,
(5) For all $t \geq 5$ a $LS[2](t,\ 2^{t-5} \cdot 7 - 1,\ 2^{t-5} \cdot (7 + 8u) - 2)$ exists for all positive numbers u,
(6) For all $t \geq 6$ a $LS[2](t,\ 2^{t-6} \cdot 9 - 1,\ 2^{t-6} \cdot 24 - 2)$ exists,
(7) For all $t \geq 6$ a $LS[2](t,\ 2^{t-6} \cdot 9 - 1,\ 2^{t-6} \cdot 25 - 2)$ exists,
(8) If $t \geq 4$ and $a_i \in \{1, 2\}$, $0 \leq i \leq t - 5$, then there exists a $LS[3](t,\ 6 \cdot 3^{t-4} + \sum_{i=0}^{t-5} a_i 3^i,\ \frac{1}{2}(3^{t-4} \cdot 31 - 3))$.
(9) For every prime p and $t \geq 1$ there exists for $a_i \in \{1, 2, \ldots, p - 1\}$, $0 \leq i \leq t-2$, a $LS[p](t,\ p^{t-1} \cdot 2 + \sum_{i=0}^{t-2} a_i p^i,\ (p^{t+1} + p^t - p^{t-1} - p)/(p - 1))$.

PROOF. As above we use Theorem 8 together with the following large sets of t-designs which are listed in [16] as starting sets:

(1) LS[2](4, 5, 12),
(2) LS[2](4, 5, 28),
(3) LS[2](4, 5, 44),
(4) LS[2](4, 5, 128),
(5) LS[2](5, 6, 5 + 8u),
(6) LS[2](6, 8, 22),
(7) LS[2](6, 8, 23),
(8) LS[3](4, 6, 14) and Theorem 4,
(9) LS[p](1, 2, p + 1) for every prime p and Theorem 4. □

The infinite family (9) in the above theorem stems from the fact that there is a LS$[n](1, k, v)$ and $n = \binom{v-1}{k-1}$ holds if and only if k divides v, see [16]. It is easy to see that for $k \leq 2v$, n is prime if and only if $k = 2$. It follows that the infinite families (9) are the only infinite families which can be constructed from this family of large sets of t-designs and the help of Theorem 4.

4 Results

The following table contains all presently known t-(v, k, λ) designs with $t \geq 7$ and $v \leq 40$. Beside the parameter sets of the designs in the first column, the second column of the table lists the construction method. If there is only the name of a group, it was found by computer search applying the Kramer-Mesner method [15] with this group as prescribed automorphism group. Every +-sign behind the name of a group means adding of one fixpoint. TvT means construction with Theorem 1, Alltop means construction via Theorem 3 and Ajoodani-Namini means use of Theorem 4. In the third column the number of isomorphism types is listed whenever it is known. If there is no reference then the designs have not yet been published elsewhere to the best of our knowledge. The 7 and 8 designs with automorphism group $Sp(6, 2)_{36}$ have been found in collaboration with I. Suleiman. The electronic version of the Atlas of finite simple groups maintained by R. Wilson et al. [9] was of great help getting generators for this group. Generators for this and many other groups may also be obtained from [11].

Parameter	construction method	isom. types	
7-(20,10,116)	$\mathrm{PSL}(2, 19)$	3	[6]
7-(20,10,124)	$\mathrm{PSL}(2, 19)$	1	[6]
7-(20,10,126)	$\mathrm{PSL}(2, 8) \times C_2 + +$	4	
7-(20,10,134)	$\mathrm{PSL}(2, 19)$	10	[6]
7-(22,11,315)	$\mathrm{PSL}(2, 19) + +$		
7-(22,11,630)	$\mathrm{PSL}(2, 19) + +$		
7-(24,8,4)	$\mathrm{PSL}(2, 23)$	1	[5,7]
7-(24,8,5)	$\mathrm{PSL}(2, 23)$	138	[5,7]
7-(24,8,6)	$\mathrm{PSL}(2, 23)$	≥ 132	[5,7]
7-(24,8,7)	$\mathrm{PSL}(2, 23)$	≥ 126	[5,7]
7-(24,8,8)	$\mathrm{PSL}(2, 23)$	≥ 63	[5,7]
7-(24,8,8)	$\mathrm{PGL}(2, 23)$	≥ 4	[6]

Parameter	construction method	isom. types	
7-(24,9,40)	$PGL(2,23)$	113	[5,7]
7-(24,9,48)	$PGL(2,23)$	≥ 2827	[5,7]
7-(24,9,64)	$PGL(2,23)$	≥ 15335	[5,7]
7-(24,10,240)	$PGL(2,23)$		[6]
7-(24,10,320)	$PGL(2,23)$	≥ 2	[6]
7-(25,9,45)	TvT 7-(24,8,5) $\cup$ 7-(24,9,40)		[5,7]
7-(25,9,54)	TvT 7-(24,8,6) $\cup$ 7-(24,9,48)		[5,7]
7-(25,9,72)	TvT 7-(24,8,8) $\cup$ 7-(24,9,64)		[5,7]
7-(25,10,288)	TvT 7-(24,9,48) $\cup$ 7-(24,10,240)		[6]
7-(25,10,384)	TvT 7-(24,9,64) $\cup$ 7-(24,10,320)		[6]
7-(26,8,6)	$PGL(2,25)$	7	[5,7]
7-(26,9,54)	$P\Gamma L(2,25)$	3989	[5,7]
7-(26,9,63)	$P\Gamma L(2,25)$	37932	[5,7]
7-(26,9,81)	$P\Gamma L(2,25)$		[5,7]
7-(26,10,342)	TvT 7-(25,9,54) $\cup$ 7-(25,10,288)		[6]
7-(26,10,456)	TvT 7-(25,9,72) $\cup$ 7-(25,10,384)		[6]
7-(26,11,1176)	$P\Gamma L(2,25)$		
7-(26,11,1356)	$P\Gamma L(2,25)$		
7-(26,11,1536)	$P\Gamma L(2,25)$		
7-(26,11,1716)	$P\Gamma L(2,25)$		
7-(26,11,1896)	$P\Gamma L(2,25)$		
7-(26,11,1926)	$P\Gamma L(2,25)$		
7-(26,12,5796)	$P\Gamma L(2,25)$		
7-(26,13,7728)	$PSL(2,23)++$ Alltop		
7-(26,13,7854)	$PSL(2,23)++$ Alltop		
7-(26,13,8190)	$PSL(2,23)++$ Alltop		
7-(26,13,8316)	$PSL(2,23)++$ Alltop		
7-(26,13,8652)	$PSL(2,23)++$ Alltop		
7-(26,13,8778)	$PSL(2,23)++$ Alltop		
7-(26,13,9114)	$PSL(2,23)++$ Alltop		

Parameter	construction method	isom. types
7-(26,13,9240)	$PSL(2,23)++$ Alltop	
7-(26,13,9702)	$PSL(2,23)++$ Alltop	
7-(26,13,10164)	$PSL(2,23)++$ Alltop	
7-(26,13,10500)	$PSL(2,23)++$ Alltop	
7-(26,13,10626)	$PSL(2,23)++$ Alltop	
7-(26,13,10962)	$PSL(2,23)++$ Alltop	
7-(26,13,11088)	$PSL(2,23)++$ Alltop	
7-(26,13,11424)	$PSL(2,23)++$ Alltop	
7-(26,13,11550)	$PSL(2,23)++$ Alltop	
7-(26,13,11886)	$PSL(2,23)++$ Alltop	
7-(26,13,12012)	$PSL(2,23)++$ Alltop	
7-(26,13,12348)	$PSL(2,23)++$ Alltop	
7-(26,13,12474)	$PSL(2,23)++$ Alltop	
7-(26,13,12810)	$PSL(2,23)++$ Alltop	
7-(26,13,12936)	$PSL(2,23)++$ Alltop	
7-(26,13,13272)	$PSL(2,23)++$ Alltop	
7-(26,13,13398)	$PSL(2,23)++$ Alltop	
7-(27,9,60)	TvT 7-(26,8,6) $\cup$ 7-(26,9,54)	[5,7]
7-(27,10,240)	$P\Gamma L(2,25)+$	[5,7]
7-(27,10,540)	$P\Gamma L(2,25)+$	[5,7]
7-(28,10,630)	$P\Gamma L(2,27)$	≥ 100 [6]
7-(29,10,420)	$P\Gamma L(2,27)+$	
7-(29,11,2130)	$P\Gamma L(2,27)+$	
7-(29,11,3465)	$P\Gamma L(2,27)+$	
7-(30,9,93)	derived 8-(31,10,93)	[4]
7-(30,9,100)	derived 8-(31,10,100)	[4]
7-(30,9,105)	$P\Gamma L(2,27)++$	[6]
7-(30,9,112)	$P\Gamma L(2,27)++$	[6]
7-(30,10,651)	residual 8-(31,10,93)	[4]
7-(30,10,700)	residual 8-(31,10,100)	[4]

Parameter	construction method	isom. types
7-(30,15,245157)	6-(14,7,4) and Ajoodani-Namini	[1]
7-(31,10,480)	PSL$(3,5)$	
7-(31,10,744)	reduced 8-(31,10,93)	[4]
7-(31,10,800)	reduced 8-(31,10,100)	[4]
7-(33,8,10)	PΓL$(2,32)$	4996426 [3,30]
7-(33,9,65)	PΓL$(2,32)$	
7-(33,9,80)	PΓL$(2,32)$	
7-(33,9,85)	PΓL$(2,32)$	
7-(33,9,100)	PΓL$(2,32)$	
7-(33,9,105)	PΓL$(2,32)$	
7-(33,9,120)	PΓL$(2,32)$	
7-(33,9,125)	PΓL$(2,32)$	
7-(33,9,140)	PΓL$(2,32)$	
7-(33,9,145)	PΓL$(2,32)$	
7-(33,9,160)	PΓL$(2,32)$	
7-(33,10,600)	PΓL$(2,32)$	
7-(33,10,720)	PΓL$(2,32)$	
7-(33,10,840)	PΓL$(2,32)$	
7-(33,10,880)	PΓL$(2,32)$	
7-(34,9,135)	PΓL$(2,32)$+	
7-(34,9,171)	PΓL$(2,32)$+	
7-(34,10,945)	TvT 7-(33,9,105) $\cup$ 7-(33,10,840)	
7-(35,10,1260)	derived 8-(36,11,1260)	
7-(35,11,7875)	residual 8-(36,11,1260)	
7-(36,11,3360)	$Sp(6,2)_{36}$	
7-(36,11,4200)	$Sp(6,2)_{36}$	
7-(36,11,4536)	$Sp(6,2)_{36}$	
7-(36,11,4935)	$Sp(6,2)_{36}$	
7-(36,11,5040)	$Sp(6,2)_{36}$	
7-(36,11,5271)	$Sp(6,2)_{36}$	

Parameter	construction method	isom. types
7-(36,11,5376)	$Sp(6,2)_{36}$	
7-(36,11,5775)	$Sp(6,2)_{36}$	
7-(36,11,5880)	$Sp(6,2)_{36}$	
7-(36,11,6111)	$Sp(6,2)_{36}$	
7-(36,11,6216)	$Sp(6,2)_{36}$	
7-(36,11,6615)	$Sp(6,2)_{36}$	
7-(36,11,6720)	$Sp(6,2)_{36}$	
7-(36,11,7056)	$Sp(6,2)_{36}$	
7-(36,11,7455)	$Sp(6,2)_{36}$	
7-(36,11,7560)	$Sp(6,2)_{36}$	
7-(36,11,7791)	$Sp(6,2)_{36}$	
7-(36,11,7896)	$Sp(6,2)_{36}$	
7-(36,11,8295)	$Sp(6,2)_{36}$	
7-(36,11,8400)	$Sp(6,2)_{36}$	
7-(36,11,8631)	$Sp(6,2)_{36}$	
7-(36,11,8736)	$Sp(6,2)_{36}$	
7-(36,11,9135)	reduced 8-(36,11,1260)	
7-(36,11,9240)	$Sp(6,2)_{36}$	
7-(36,11,9471)	$Sp(6,2)_{36}$	
7-(36,11,9576)	$Sp(6,2)_{36}$	
7-(36,11,9975)	$Sp(6,2)_{36}$	
7-(36,11,10080)	$Sp(6,2)_{36}$	
7-(36,11,10311)	$Sp(6,2)_{36}$	
7-(36,11,10416)	$Sp(6,2)_{36}$	
7-(36,11,10815)	$Sp(6,2)_{36}$	
7-(36,11,10920)	$Sp(6,2)_{36}$	
7-(36,11,11151)	$Sp(6,2)_{36}$	
7-(36,11,11655)	$Sp(6,2)_{36}$	
7-(36,11,11760)	$Sp(6,2)_{36}$	
7-(39,10,1440)	derived 8-(40,11,1440)	[8]

Parameter	construction method	isom. types	
7-(39,11,10440)	residual 8-(40,11,1440)		[8]
7-(40,10,560)	$\mathrm{PSL}(4,3)$		
7-(40,10,1008)	$\mathrm{PSL}(4,3)$		
7-(40,10,1208)	$\mathrm{PSL}(4,3)$		
7-(40,10,1296)	$\mathrm{PSL}(4,3)$		
7-(40,10,1568)	$\mathrm{PSL}(4,3)$		
7-(40,10,1656)	$\mathrm{PSL}(4,3)$		
7-(40,10,2304)	$\mathrm{PSL}(4,3)$		
7-(40,10,2504)	$\mathrm{PSL}(4,3)$		
7-(40,11,11880)	reduced 8-(40,11,1440)		[8]
8-(31,10,93)	$\mathrm{PSL}(3,5)$	138	[4]
8-(31,10,100)	$\mathrm{PSL}(3,5)$	1658	[4]
8-(36,11,1260)	$Sp(6,2)_{36}$		
8-(40,11,1440)	$\mathrm{PSL}(4,3)$	$\geq$ 100000	[8]

The latest results on t-designs constructed by DISCRETA and the program itself are located at

http://www.mathe2.uni-bayreuth.de/betten/DESIGN/d1.html .

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