# Groups in Action '96

Thurnau
October 20 - 24, 1996

SUNDAY, October 20, 1996:
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arrival of participants:

in Thurnau
Wissenschaftszentrum Thurnau,
Schloss Thurnau
Herr Friedrich,
phone 09228 / 954220

or Bayreuth (train station or university)
phone university:
0921 / 55 33 88 or
0921 / 55 36 71

phone Prof. Kerber (Bayreuth)
0921 / 68009

19:30 lunch in Thurnau (Pizzeria am See)

MONDAY, October 21:
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9:00 (60 min) Modular homology in the Boolean Algebra and in Projective Space,
J Siemons (Norwich, UK) J.Siemons@uea.ac.uk

10:00 (30 min break)

10:30 (45 min incl short break) Berechnung endlicher aufloesbarer Quotienten
Herbert Brueckner, Aachen herbert@willi.math.rwth-aachen.de

11:15 (45 min) Tactical decompositions and some configurations $n_4$
Dieter Betten, Kiel betten@math.uni-kiel.de

12:00 break

14:30 (50 min incl. short break) New Results about Geometric Codes
David Glynn, Christchurch, New Zealand glynn@math.uni-kiel.de

15:20 (40 min) Computing maximal subgroups of space groups in GAP,
Bettina Eick, Aachen Bettina.Eick@Math.RWTH-Aachen.DE

16:00 break (20 min)

16:20 (40 min) Maximal finite matrix groups
Gabriele Nebe, Aachen gabi@willi.math.rwth-aachen.de

17:00 (30 min) Zyklische Charaktere der Symmetrischen Gruppe
Thomas Scharf, Bayreuth haegar@btm2x2.mat.uni-bayreuth.de

18:30 small supper

TUESDAY, October 22:
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9:00 (60 min) Konstruktion transitiver Permutationsgruppen,
Alexander Hulpke, Aachen Alexander.Hulpke@Math.RWTH-Aachen.DE

10:00 (30 min break)

10:30 (40 min incl. short break) Konstruktion Diskreter Strukturen
Reinhard Laue, Bayreuth laue@btm2x2.mat.uni-bayreuth.de

11:10 (50 min) Enumerative problems in framework of group action orthogonality
Valerij Liskovets, Dresden liskov@nalw01.math.tu-dresden.de

12:00 break

14:30 (50 min incl. short break) Construction of symmetric (176,50,14) designs using subgroups of the sporadic Higman-Sims-group,
Wolfram Wirth, Mainz WOLFRAM@MAT.Mathematik.Uni-Mainz.DE

15:20 (40 min) A computer approach to the block-designs which are invariant with respect to a prescribed permutation group
Christian Pech, Dresden pech@bgumail.bgu.ac.il pech@virgo.sax.de

16:00 30 min break

16:30 (30 min incl short break) Simple $t$-Designs with large $t$: A Status Report
Alfred Wassermann, Bayreuth Alfred.Wassermann@uni-bayreuth.de

17:00 (30 min) The construction of configurations — The method of Martinetti and modern methods,
Harald Gropp, Heidelberg d12@ix.urz.uni-heidelberg.de

17:30

WEDNESDAY, October 23:
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9:00 (60 min) Classifying Line-Transitive Finite Linear Spaces?,
Alan Camina (Norwich, UK) A.Camina@qmw.ac.uk

10:00 (30 min break)

10:30 (30 min incl. short break) Merging of graphs
Thomas Wieland thomas_w@btm2x2.mat.uni-bayreuth.de

11:00 (30 min) Graph generation
Thomas Gruener thomas_g@btm2x2.mat.uni-bayreuth.de

12:00 long break
possibility for a trip to Bayreuth: Festspielhaus, Oper or Haus Wahnfried

16:30 (30 min incl. short break) Enumeration, construction and random generation of block codes
Harald Fripertinger, Graz fripert@balu.kfunigraz.ac.at

17:00 (30 min incl. short break) Erzeugung regularer Graphen
Markus Meringer markus@btm2x2.mat.uni-bayreuth.de

17:30 (30 min) The linear spaces on $\leq$ 12 points
Anton Betten, Bayreuth Anton.Betten@uni-bayreuth.de

18:00

THURSDAY, October 24:
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9:00 Kerber, Bayreuth kerber@btm2x2.mat.uni-bayreuth.de

9:15 (55 min) The Classification of p-Groups by Coclass
Axel Schneider, SchneidA@Maths.UNSW.Edu.AU

10:10 30 min break

10:40 (30 min) Symmetry Groups of Finite Generalized Quadrangles.,
Dirk Hachenberger, Augsburg Dirk.Hachenberger@Math.Uni-Augsburg.DE

11:10 finished !!!

# The linear spaces on $\leq$ 12 points

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**Abstract.**
The number of linear spaces on $v$ points LIN$(v)$ is SLOANE's Sequence no. A001200 [?] (encyclopedia number M0726 [?]): $1, 1, 1, 2, 3, 5, 10, 24, 69, 384, 5250, 232929$ for $v = 0, 1, \ldots, 11$.
It is the aim of this talk to extend this sequence by the term LIN(12) = 28872973.
The construction of geometries has a preprocessing phase where possible parameters of the geometries are computed algebraically. So, the geometries are enumerated according to their line type. If the subcases by line type are too big for construction one specifies the point types and the distribution of points of different type. If these subcases by point types are still too large, refined line cases are introduced.
This method of varying depth of parameter precomputation allows to react in a flexible way to different grades of difficulty in the construction of linear spaces.
The TDO-method of BETTEN and BRAUN [?] is used as a tool for classification of the geometries. The isomorphism problem is solved by using canonical forms of the geometries. The canonical form respects the TDO invariant and can be computed efficiently if the geometry already has a good decomposition by its TDO.
AMS subject classification: 05B25, 05B30, 51E99
DIETER BETTEN, MATHIAS BRAUN: A tactical decomposition for incidence structures. *Ann. Disc. Math.* **52** (1992), 37-43.
NEAL J.A. SLOANE: A handbook of integer sequences. *Academic Press* 1973.
NEAL J.A. SLOANE: Database of integer sequences (WWW version)
`http://netlib.att.com/math/sloane/doc/eisbt0.html`

# Can we classify line-transitive finite linear spaces

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## 1 Introduction

In this talk I want discuss how far can we progress in trying to classify pairs of linear spaces with their automorphism groups.
Let us start with the definition of linear spaces.

**Definition 1** *A* Finite Linear Space, $\mathcal{S}$ *, is a finite set of points* $\mathcal{P}$ *and a set of subsets,* $\mathcal{L}$ *called lines, such that every pair of points is in a unique line.*

The number of points will be denoted by $v$ and when each line has the same size this size will be denoted by $k$. When this happens we call the space *regular* and denote the number of lines through a point by $r$. In this situation the spaces is a block design or Steiner system. The next thing we need is a definition of automorphism.

**Definition 2** *Let* $\mathcal{S}$ *be a linear space. An automorphism* $g$ *of* $\mathcal{S}$ *is a permutation of* $\mathcal{P}$ *which induces a permutation on the lines.*

NOTE Any permutation of $\mathcal{S}$ induces two permutations one on the points and one on the lines. These will be denoted by $_P$ and $_L$ respectively. The full automorphism group of a linear space $\mathcal{S}$ will be denoted by Aut($\mathcal{S}$). Any subgroup of Aut($\mathcal{S}$) will be called an *automorphism group* of $\mathcal{S}$.

**Question 1** *Given a pair* $(\mathcal{S}, G)$ *with* $G \subseteq Aut(\mathcal{S})$ *what can we say about them?*

Here is a trivial example.

**Theorem 1** *Let* $\mathcal{S}$ *be a finite linear space and assume that Aut(*$\mathcal{S}$*) is* 3*-transitive on points then* $\mathcal{S}$ *is either a line or every line has exactly two points.*

The proof of this is trivial. Let us from now on assume that linear spaces have at least one line of size larger than three and at least two lines.
The next result is somewhat harder and was not proved until the classification of finite simple groups.

**Theorem 2** *(W.M. Kantor (1975)) Let $\mathcal{S}$ be a finite linear space and assume that $Aut(\mathcal{S})$ is 2-transitive on points then the pair $(\mathcal{S}, Aut(\mathcal{S}))$ is known.*

In this proof the classification is used to list the known 2-transitive groups and from their properties a list of the candidates and the spaces are found.
The next stage was to look at the situation when the automorphism group was flag-transitive. This lead to the following result.

**Theorem 3** *Let $\mathcal{S}$ be a finite linear space and assume that $Aut(\mathcal{S})$ is flag-transitive then the pair $(\mathcal{S}\ \mathcal{S}, Aut(\mathcal{S}))$ is known.*

This was proved by F. Buekenhout, A. Delandtsheer, J. Doyen, P. B. Kleidman, M. W. Liebeck, and J. Saxl and was announced in 1990. This classification is not quite as precise as the statement suggests nor have the proofs been published. But what are the ingredients of the proof.

- The group is primitive.
- The socle of the group is either elementary abelian or simple.

It is now possible to proceed as in the 2-transitive case, except in that case the doubly transitive permutation groups had already been classified. However there are rather more groups to investigate and the elementary abelian case is not completely classified in the sense that a complete listing of groups and spaces is not available. Work by Kantor and Williams (1996 JCTA) illustrates this quite well. If we now assume that this classification is complete or at least as complete as it is going to be, the next step would be to look at the following question:-

**Question 2** *Let $\mathcal{S}$ be a finite linear space and assume that $Aut(\mathcal{S})$ is line-transitive can we classify the pair $(\mathcal{S}, Aut(\mathcal{S}))$?*

# 2 Imprimitivity

Since the first of the ingredients does not neccessarily hold it seems unlikely that such a classification is possible. However there is some hope. The first is a special case of a result of A. Delandtsheer and J. Doyen:-

**Theorem 4** *Let $\mathcal{S}$ be a finite linear space with a line-transitive automorphism group $G$. Assume that $G$ is not primitive on points. Let v=cd, where c is the size of the sets of imprimitivity. Then there exist natural numbers $x$ and $y$ such that*

$$c = \frac{\binom{k}{2} - x}{y} \tag{1}$$

$$d = \frac{\binom{k}{2} - y}{x} \tag{2}$$

*are natural numbers.*

In particular for a given value of $k$ there are only finitely many finite linear spaces which can have line-transitive point-imprimitive automorphism groups. This gives rise to the following question:-

**Question 3** *Is it possible to classify pairs* $(\mathcal{S}, G)$ *where* $G$ *is a line-transitive point-imprimitive automorphism group of* $\mathcal{S}$.

A step towards solving this is the following:-

**Theorem 5** *(S. Mishcke(1996)) Let* $\mathcal{S}$ *be a linear space. Assume that* $\mathcal{S}$ *has an automorphism group* $G$ *which is line-transitive and point-imprimitive with* $k < 9$. *Then* $\mathcal{S}$ *is one of the following:-*

**(a)** *A projective plane of order* $4$ *or* $7$,

**(b)** *One of* $2$ *linear spaces with* $v = 91$ *and* $k = 6$,

**(c)** *One of* $467$ *linear spaces with* $v = 729$ *and* $k = 8$.

*In all cases the full automorphism group* $Aut(\mathcal{S})$ *is known.*

The result Theorem 5 (c) is due to a combination of work by Nickel, Niemeyer, O'Keefe, Penttila and Praeger. Is this a sporadic example due to the numbers involved or will this keep happening? The examples in Theorem 5 (a) keeps repeating. For any projective plane of order $n$ with a Singer cycle whose order is not prime will give an example. Thus we pose the following two questions:-

**Question 4** *Let* $\mathcal{L}$ *be a classical projective plane of order* $q$. *What are the possible automorphism groups which act line-transitively but not point-primitively?*

Note that this is really a question about PGL(3,q).

**Question 5** *What happens for the non-classical projective planes?*

Here are atwo theorems, with Cheryl Praeger, which are relevant to the imprimitive situation.

**Theorem 6** *Let* $\mathcal{L}$ *be a finite linear space and let* $G$ *be a line-transitive automorphism group of* $\mathcal{L}$. *Let* $N$ *be a normal subgroup of* $G$. *Then* $N$ *acts faithfully on each of its point orbits.*

**Corollary 1** *Let* $\mathcal{L}$ *be a finite linear space and let* $G$ *be a line-transitive automorphism group of* $\mathcal{L}$. *If* $O_2(G) \neq 1$ *then* $G$ *is flag- transitive.*

**Theorem 7** *Let* $G$ *be a line-transitive automorphism group of a finite linear space* $\mathcal{L}$. *Then* $G$ *has at most one non-abelian minimal normal subgroup.*

# 3 Primitivity

However what happens if we assume that the group acts primitively on points.

**Theorem 8** *(A. Camina(1996)) Let $\Gamma$ be a line-transitive, point-primitive automorphism group of a linear space $\mathcal{S}$ . Then the socle of $\Gamma$ is either elementary Abelian or simple.*

Using this F. Spezia has shown that there are no examples with the socle is a sporadic group. Consideration of the remaining simple groups is beginning but is considerably harder than in the flag-transitive situation. One of the reasons for this is that if $G$ acts flag-transitively the stabilizer of a point is relatively large however this is not the case without this condition.

# 4 Methods

Let me mention a very useful counting lemma due to Huw Davies.

**Lemma 1** *Let $\mathcal{S}$ be a regular linear space and let $g \neq 1$ be an automorphism of $\mathcal{L}$ with order $p$ where $p$ is prime. Then $|Fix_{\mathbf{P}}(g)| \leq r+k-p-1$. Further $|Fix(g)| \leq r$ unless every point lies on a fixed line.*

Note that $\mathrm{Fix}(g)$ is the set of fixed points of $g$.
Thus we are naturally led into examining the properties of fixed point sets of automorphisms.

**Lemma 2** *Let $G$ be a line-transitive automorphism group of a linear space $\mathcal{L}$ . Let $\ell$ be a line and let $H \subset G_\ell$. Assume that $H$ satisfies the following two conditions*

- *i) $|Fix(H) \cap \ell| > 1$ and*
- *ii) if $K \leq G_\ell$ and $|Fix(H) \cap \ell| > 1$ and $K$ is conjugate to $H$ in $G$ then $K$ is conjugate to $H$ in $G_\ell$.*

*Then either*

- *i) $Fix(H) \subseteq \ell$ or*
- *ii) the induced linear space on $Fix(H)$ is a regular linear space and $N_G(H)$ acts line-transitively on the space.*

**Lemma 3** *Let $G$ be a line-transitive automorphism group of a linear space $\mathcal{L}$ . Let $\ell$ be a line and let $p$ be a prime. Assume that*

- *i) if $R$ is a $p$-subgroup of $G_\ell$ then $|Fix(R) \cap \ell| > 0$ implies that $|Fix(R) \cap \ell| > 1$ and*
- *ii) there exists a $p$-subgroup $S$ of $G_\ell$ so that $Fix(S) = \ell$.*

*Then $G$ is flag-transitive.*

**Lemma 4** *Let $\mathcal{S}$ be a regular linear space and let $G$ be an automorphism group of $\mathcal{S}$ which is line-transitive. If $k \mid v$ then $G$ is flag-transitive.*

**Lemma 5** *Let $\mathcal{L}$ be a line-transitive linear space and let $g$ be an automorphism of order $2$. If $g$ acts fixed-point-freely on points then $\mathcal{L}$ is flag-transitive.*

This is important in the way that it enables us to eliminate variou possibilities. It shows, for example, that if $G$ is point-primitive but not flag-transitive that every element of order 2 has a conjugate in the stabiliser of a point. If $G$ is simple this means that certain classes of maximal subgroup are not candidates for the the stabilisers of points in an hoped for representation.
Here is one last question

**Question 6** *Let $\mathcal{L}$ be a finite linear space and let $G$ be a line-transitive automorphism group of $\mathcal{L}$ . If $v$ is even does this imply that $G$ is flag-transitive.*

# Literatur


[1] W. M. Kantor. Homogeneous designs and geometric lattices. *Journal of Combinatorial Theory, A*, 38:66–74, 1985.

[2] W. M. Kantor and M. E. Williams New flag-transitive affine planes of even order. Jour. Combin. Theory, Series A 64 (1996) 1-13.

[3] F. Buekenhout, A. Delandtsheer, J. Doyen, P. B. Kleidman, M. W. Liebeck, and J. Saxl. Linear-spaces with flag-transitive automorphism groups, *Geom. Dedicata* 36 (1990) 89–94.

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[6] C. M. O'Keefe, T. Penttila, and C. E. Praeger. Block-transitive point-imprimitive designs with $\lambda$=1. *Discrete Math.* 115 (1993) 231–244.

[7] W. Nickel, A. Niemeyer, C. M. O'Keefe, T. Penttila, and C. E. Praeger. The block-transitive point-imprimitive $2-(729,8,1)$ designs. *Applied Algebra in Eng. Comm. and Science*, 3 (1992) 47–61.

[8] A. R. Camina and C. E. Praeger. Line-transitive automorphism groups of linear spaces *Bull. London Math. Soc.* 25 (1993) 309–315.

[9] A. R. Camina The socle of automorphism groups of linear spaces. *Bull. London Math. Soc.* 28 (1996) 269–272

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[11] A. R. Camina and J. Siemons. Block transitive automorphisms of 2-(v, k, 1) block designs. *Journal Combinatorial Theory A*, 51 (1989) 268–276.

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# Symmetry Groups of Finite Generalized Quadrangles.

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**Abstract.** Finite Geometries are often studied under the assumption of certain automorphism groups, whence examples can be constructed by coordinatizing the underlying object in terms of the group acting on it. This is also the case for class of geometries, I am considering in my talk, namely Finite Generalized Quadrangles (GQ). GQs were introduced as particular Generalized Polygones by J. Tits (1959), and have attained much interest since then. The standard reference is the monography of S. Payne and J. Thas (1984). Most of the presently known families of GQs can be represented as a group coset geometry with respect to a certain group of automorphisms. Necessarily, the subgroup lattice of the coordinatizing group has to satisfy a particular combinatorial property (it has to admit a so-called Kantor family or 4-gonal family). We report on recent results on groups admitting a Kantor family under the assumption that the family contains some particular symmetries of the coordinatized quadrangle. One of our main results is that the parameters of a Skew Translation Generalized Quadrangle (STGQ) necessarily are powers of the same prime. This is of interest since all the nine new families of GQs found after 1980 belong to the class of STGQ. Moreover, it settles the prime power conjecture of W. Kantor for the class of STGQ.

# Simple $t$-Designs with large $t$: A Status Report

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**Abstract.**
In the last two years considerable progress in the construction of simple $t$-designs with $t \geq 6$ has been made. All known examples with reasonable parameters have been found by applying a group action via the theorem of Kramer and Mesner together with efficient algorithms for finding $\{0,1\}$-solutions of linear equation systems.
Details of the algorithm together with an overview of the results are given.

# New Results about Geometric Codes

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**Abstract.**
Consider the collection of all $i$-dimensional subspaces of $PG(n,q)$, the finite projective space of dimension n over the finite field GF(q). The geometric code $C_i(n,q)$ is then the vector space of all functions from the points of $PG(n,q)$ to $GF(q)$, such that the sum over all points of any subspace of dimension $i$ is zero. This code is dual to $D_i(n,q)$, which is generated by the characteristic functions of the subspaces of dimension $m$. We shall explain how these codes are formed using algebraic functions: there are natural bases using monomials. As an example, we can determine precisely when two of the codes are orthogonal. Basically, $C_i$ is orthogonal to $C_j$ (or equivalently $C_i$ is contained in $D_j$) when $q = p^h$, and $h$ is small enough, (where $p$ is the characteristic of $GF(q)$). This is joint work with Baskhar Bagchi of the Indian Statistical Institute in Bangalore.

# Maximal Finite Matrix Groups

Gabriele Nebe

**Abstract.**
Teilweise mit Herrn Plesken habe ich die maximal endlichen rationalen Matrixgruppen bis zum Grad 31 klassifiziert, indem ich ein Vertretersystem der Konjugiertenklassen maximal endlicher irreduzibler Untergruppen von GLn(Q) (n¡=31) bestimmt habe. Inzwischen habe ich Q durch eine total definite Quaternionenalgebra ersetzt und eine aehnliche Klassifikation erzielt. In meinem Vortrag werde ich mich auf die Gestalt dieser Matrixgruppen konzentrieren.

# Enumerative problems in framework of group action orthogonality

Valerij Liskovets

**Abstract.**
The count of orbits of a group acting on a set may be facilitated considerably by knowledge of another action orthogonal to the first one. (Two group actions are called mutually orthogonal if any non-identity element has a fixed point in at most one of them.) This opportunity is characteristic for various combinatorial, topological and algebraic objects which can be modelled by transitive tuples of permutations. Numerous open problems are posed and discussed.

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